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ON SINGULAR CHARACTERISTIC INITIAL VALUE PROBLEMS WITH UNIQUE S--ETC(U)

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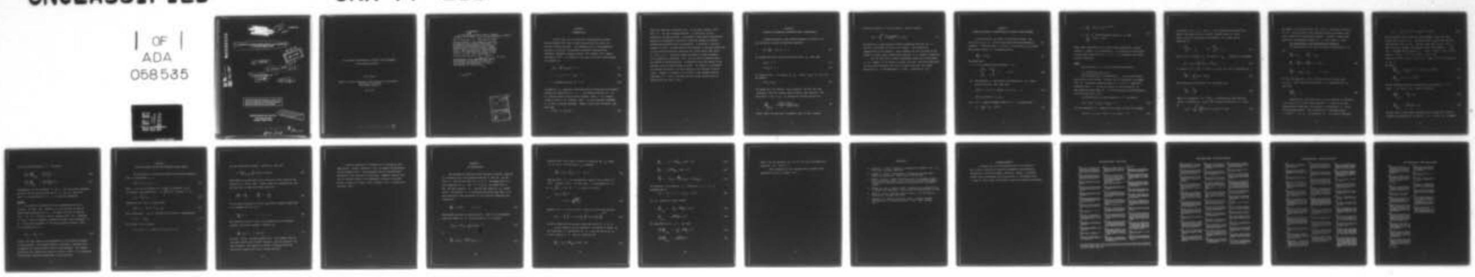

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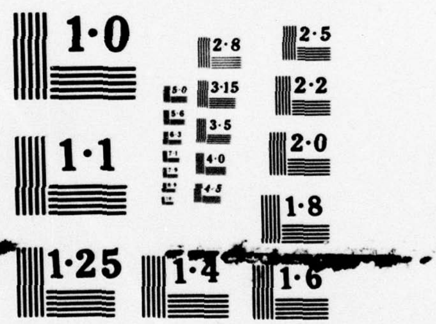

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10 Marc Mangel

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ON SINGULAR CHARACTERISTIC INITIAL VALUE PROBLEMS

WITH UNIQUE SOLUTIONS

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ABSTRACT

We consider a special class of characteristic first-order initial value problems are considered. $F(x^1,x^2,\psi,p_1,p_2) = 0$. The initial value problem arises in the asymptotic solution of parabolic and elliptic equations. The problem is characterized by a singular, characterisitc initial manifold. Namely, initial data is given on a characteristic curve. The characteristic curve is also singular in that there is a point on the initial manifold where $F_{p_1}^2 + F_{p_2}^2 = 0$. the equation

~~We prove~~ that such problems have unique solutions,* The theorem also has an interesting interpretation in terms of the calculus of variations.

* is proven.

SECTION 1

INTRODUCTION

In this note, we will discuss a characteristic initial value problems (IVP) that is more singular than the usual IVP, but has a unique solution. The problems arise in the asymptotic solution of elliptic and parabolic equations connected with stochastic dynamical systems (Ludwig, 1975; Mangel, 1977; Mangel and Ludwig, 1977). An example of the type of initial value problem of interest is:

$$b^i\psi_i - \frac{a^{ij}}{2}\psi_i\psi_j f(\psi) = 0 \tag{1}$$

$$\psi = \psi_o \text{ on } S \; ; \; f(\psi_o) = 0 \tag{2a}$$

$$S \text{ parameterized by } \dot{x}^i = b^i \tag{2b}$$

In equation (1), subscripts indicate partial derivatives and repeated indices are summed from 1 to n. The problem posed by (1) - (2) is a characteristic initial value problem, since S is a characteristic curve of (1) (Courant, 1962). In the stochastic problems, S is also a singular manifold. Namely, there exits one point $Q \in S$, such that

$$b^i(Q) = 0 \text{ for all } i \,. \tag{2c}$$

Under the additional condition (2c), it was shown (Mangel, 1977), that (1) - (2) has a unique solution. In this note, we will generalize the above result. An analogy to the partial differential equation is presented in section 2, where we discuss an ordinary differential equation with a singularity. In section 3, the main theorem is given. In the proof of the theorem, we use the singularity of the initial manifold to construct the unique normal derivative of $\psi(x)$ on S. Once the normal derivative is known uniquely, it is possible to calculate $\psi(x)$ off the initial manifold using the method of characteristics. A comparison with the theory of the standard initial value problem is given. In section 4, we discuss the theorem of section 3 in the setting of the calculus of variations. Finally, in section 5, we return to the example given by equations (1) - (2a,b), and show how the unique solution can be obtained for the case $f(\psi)=\psi$.

SECTION 2

ORDINARY DIFFERENTIAL EQUATIONS WITH A SINGULARITY

A simple analogy to the problem presented in section 1 is the following ordinary differential equation:

$$b(x)\,\frac{d\psi}{dx} + f(x,\ \psi) = 0 \tag{3}$$

We assume that there exists exactly one point x_p such that

$$b(x_p) = 0 \tag{4}$$

$$b'(x_p) \neq 0\ . \tag{4'}$$

We require that ψ is regular at x_p . Since $b(x_p) = 0$, for ψ to be regular:

$$f(x_p,\ \psi(x_p)) = 0 \tag{5}$$

We assume that (5) defines $\psi(x_p)$ uniquely. We note that the regularity condition imposes initial data on the equation. The derivative $d\ /dx$ at x_p is obtained by differentiating (3):

$$\left.\frac{d\psi}{dx}\right|_{x=x_p} = \frac{-f_x(x_p,\ \psi(x_p))}{b'(x_p) + f_\psi(x_p, \psi(x_p))} \tag{6}$$

Hence, under the additional assumption that (5) has a unique

solution, equation (3) has the unique, regular solution

$$\psi(x) = \int_{x}^{x_p} \frac{f(t, \psi(t))dt}{b(t)} + \psi(x_p) \tag{7}$$

We obtained a unique solution without specifying any data for ψ; the only requirement was that it be regular. The singularity imposes data on the equation. Many examples can also be found in the theory of second order differential equations (e.g., Bessel's equation).

We will show that a similar phenomenon occurs with the problem (1, 2). Namely, if we require a regular solution, then the singularity on the initial manifold will force a unique value for the normal derivative of ψ (corresponding to $d\psi/dx$ evaluated at x_p).

SECTION 3

UNIQUE SOLUTIONS OF CHARACTERISTIC INITIAL VALUE PROBLEMS

In this section, we prove the following theorem. Let $F(x^1, x^2, \psi, \psi_1, \psi_2) = 0$ be one first order partial differential equation. Let Cauchy data $\psi = \psi^0(t)$ on a manifold S, be given. (8)

The initial surface S is parametrized by

$$\frac{dx^i}{dt} = b^i(x) . \tag{9}$$

We assume that:

i) S is a characteristic manifold, i.e.,

$$\Delta = \begin{vmatrix} F_{\psi_1} & F_{\psi_2} \\ b^1 & b^2 \end{vmatrix} = 0 \qquad \text{on } S; \tag{10}$$

ii) The mainfold S has exactly one singularity, i.e., there exists one point $Q \in S$ such that

$$F^2_{\psi_1}(Q, \psi, \psi_1, \psi_2) + F^2_{\psi_2}(Q, \psi, \psi_1, \psi_2) = 0 \tag{11}$$

for all values of ψ_1, ψ_2 .

iii) Let n denote distance normal to S . We assume that

$$\text{a)} \quad \lim_{n \to 0} F_{\psi_1} = 0(n^{\alpha}) \tag{12}$$

b) $\lim\limits_{n \to 0} \; F_{\psi_1} \psi_1 \; F_{\psi_2} \psi_2 = 0(n^{\alpha})$

c) Let x^1 denote distance normal to S. Then (12)

$$\lim_{n \to 0} \; b^1(x) = 0(n^{\alpha}).$$

Under these assumptions, the initial value problem has a unique solution. Namely, it is possible to: i) calculate a unique value of the normal derivative of ψ on S ; ii) integrate the characteristic equations.

<u>PROOF</u>

We prove parts i, ii of the conclusion separately.

<u>i) Calculation of ψ_n on S</u>

As is usually done, we assume that ψ has as many smooth derivatives as are needed. Without loss of generality, we assume that x^1 represents distance normal to S and x^2 represents distance along S . We set the origin at the singularity Q. The tangential derivative of ψ can be obtained by differentiation of the initial data $\psi^0(t)$.

We differentiate (8) with respect to x^1 and obtain

$$F_{x^1} + F_{\psi}\psi_1 + F_{\psi_1}\psi_{11} + F_{\psi_2}\psi_{21} = 0 \tag{13}$$

At the singularity Q , equation (11) holds, so that (13) becomes

$$F_{x^1}(Q, \psi, \psi_1, \psi_2) + F_{\psi}(Q, \psi, \psi_1, \psi_2)\psi_1 = 0 \tag{14}$$

In equation (14), Q, ψ, and ψ_2 (the tangential derivative) are known. We assume that (14) yields a unique value for $\psi_1(Q)$. Next, we use the characteristic condition (10), which can be rewritten as

$$\frac{b^2 F_{\psi_1}}{b^1} = F_{\psi_2} \qquad \text{or} \qquad \frac{b^1 F_{\psi_2}}{b^2} = F_{\psi_1} \tag{15}$$

We use (15) to express F_{ψ_1} in terms of F_{ψ_2}. Equation (13) becomes

$$F_{x^1} + F_\psi \psi_1 + \frac{F_{\psi_2}}{b^2}(b^1\psi_{11} + b^2\psi_{21}) = 0 \tag{16}$$

In light of (9), $d/dt = b^i \, \partial/\partial x^i$, so that (16) can be rewritten as

$$\frac{d\psi_1}{dt} = \frac{-b^2}{F_{\psi_2}}\{F_{x^1} + F_\psi \psi_1\} \tag{17}$$

The characteristic condition (10) indicates that

$$\frac{F_{\psi_1}}{b^1} = \frac{F_{\psi_2}}{b^2} \tag{17a}$$

Hence, by assumption iii) b^2/F_{ψ_2} is bounded away from zero and finite. Consequently, ψ_1 will be a regular function on S, given by

$$\psi_1(t) = \int_t^{t^0} \frac{b^2(S)}{F_{\psi_2}}(F_{x^1} + F_\psi \psi_1)dS + \psi_1(Q) \tag{18}$$

We assume, as is usually done, that (18) is a meaningful solution of (17). We note that if the singularity were not present, then $\psi_1(t)$ would not be determined uniquely. Instead, we would have obtained a one-parameter family for ψ_1 (see remark 2, also).

ii) Integration of the Characteristic Equations

The characteristic (or ray) equations corresponding to (8) are

$$\frac{dx^1}{d\tau} = F_{\psi_1} \qquad \frac{dx^2}{d\tau} = F_{\psi_2}$$

$$\frac{d\psi}{d\tau} = \frac{dx^1}{d\tau}\psi_1 + \frac{dx^2}{d\tau}\psi_2 \equiv f(x, \psi) \tag{19}$$

$$\frac{dp_k}{d\tau} = -(p_k F_\psi + F_{x^k}) \qquad k = 1, 2$$

In the last equation in (19), we have used the notation that $\psi_k = p_k$. Since the initial manifold S is characteristic, we have that

$$\left.\frac{dx^1}{d\tau}\right|_S = 0 \tag{20}$$

Equation (20) indicates that it is not possible to integrate (19) to move off of the initial manifold. In order to use the characteristic equations, we construct a new manifold S_γ, which is not characteristic. The new manifold is characterized by $\psi = \psi^0(t) + \gamma$ on S_γ. We construct S_γ by a Taylor expansion:

$$S_\gamma \equiv \{(x^1, x^2) : x^1 = \gamma/\psi_1(x^2) + 0(\gamma^2)\} \tag{21}$$

We now consider the new initial value problem: solve (8) with initial data given on S_γ. We need to give initial data for the ray equations (19), on S_γ. We have shown that $\psi(0;\gamma) = \psi^0 + \gamma$, where the first argument corresponds to the ray parameter τ and the γ denotes that we are on S_γ. Also, $x^1(0,\gamma) = \gamma/\psi_1 + 0(\gamma^2)$. Clearly, $x^2(0,\gamma) = x^2$ and $\psi_2(0,\gamma) = 0(\gamma) + \psi_2^0$.
Finally, we have $\psi_1(0;\gamma) = \psi_1^0 + 0(\gamma)$, where ψ_1^0 is the value of ψ_1 on S.
We obtain

$$\left.\frac{dx^1}{d\tau}\right|_{\tau=0} = F_{\psi_1}\left(\frac{\gamma}{\psi_1}, x^2, \gamma, \psi_1^0, \psi_2\right) + 0\left(\gamma^{2+\alpha}\right) \tag{22}$$

$$\left.\frac{d\psi}{d\tau}\right|_{\sigma=0} = f(x, \gamma) + 0(\gamma^{2+\alpha}) \tag{23}$$

We now reparametrize the ray equations by introducing $\sigma = \tau\gamma^\alpha$, where α is given in (12). Hence, we obtain

$$\left.\frac{dx^1}{d\sigma}\right|_{\sigma=0} = \frac{F_{\psi_1}}{\gamma^\alpha} + o(\gamma) \tag{24}$$

$$\left.\frac{d\psi}{d\sigma}\right|_{\sigma=0} = \frac{f(x, \gamma)}{\gamma^\alpha} + o(\gamma) \tag{25}$$

We note that in some cases, equations (22, 23) may have to be reparametrized separately. We now let $\gamma \to 0$, so that S_γ collapses

into the original manifold S . We obtain

$$\lim_{\gamma\to 0} \left.\frac{dx^1}{d\sigma}\right|_{\sigma=0} = \lim_{\gamma\to 0} \frac{F_{\psi^1}}{\gamma^\alpha} \neq 0 \tag{26}$$

$$\lim_{\gamma\to 0} \left.\frac{d\psi}{d\sigma}\right|_{\sigma=0} = \lim_{\gamma\to 0} \frac{f(x,\gamma)}{\gamma^\alpha} \neq 0 \tag{27}$$

Consequently, we can calculate x^1 and ψ off the initial manifold, by using (26,27) and the equations for x^2 and p_k from (19). Thus, in a neighborhood of S , ψ is uniquely determined.

REMARKS

1. An intuitive description of the reparametrization is as follows. For each $\gamma\neq 0$, equations (19) are solved, so that we obtain rays emanating from S_γ . We denote this set of rays by $\{R_\gamma\}$. As $\gamma\to 0$, equations (26,27) indicate that $\{R_\gamma\}$ converges to $\{R_o\}$, rays that appear to emanate from the initial manifold S .

2. In the usual treatment of the Cauchy problem (Courant, 1962), it is assumed that on S

$$F^2_{\psi_1} + F^2_{\psi_2} \neq 0 . \tag{28}$$

Hence, the usual theory is not applicable to the problem discussed in this theorem. The standard theory, furthermore, indicates non-uniqueness for characteristic initial value problems. The theorem given here has extended the theory in Courant (1962). It is noteworthy that the more singular problem has a unique solution.

SECTION 4

THE VARIATIONAL SETTING AND HAMILTON-JACOBI THEORY

We now consider a first order partial differential equation that is independent of ψ

$$\tilde{F}(x^1, x^2, \phi_1, \phi_2) = 0 \tag{29}$$

Often, (29) can be obtained by a change of variables in (8). For example, the substitution $\phi = -\int^{\psi} f(S)dS$ converts (1) to

$$b^i\phi_i + \frac{a^{ij}}{2}\phi_i\phi_j = 0 \tag{30}$$

Associated with (29) is a Hamiltonian

$$H(x, p) = \tilde{F}(x^1, x^2, p_1, p_2) \tag{31}$$

and a Lagrangian $L(x, \dot{x})$ defined by the contact transformation

$$H + L = \dot{x}^i p_i \tag{32}$$

We consider a set of paths

$$C\,(t, x_o, x) = \{\mu(S):\mu(0) = x_o, \mu(t) = x\} \tag{33}$$

and the variational problem: choose $\phi(x)$ such that

$$\phi = \min_{C(t,x_o,x)} \int_0^t L(x(s)),\dot{x}(s))ds \tag{34}$$

The problem is solved when we can construct level curves of the function $\phi(x)$ (Rund, 1966). These curves are determined by solving the Euler or Hamilton-Jacobi equations:

$$\frac{dx^i}{ds} = \frac{\partial H}{\partial p_i} \qquad \frac{dp_i}{ds} = -\frac{\partial H}{\partial x^i} \tag{35}$$

In the usual formulation of such variational problems (Rund,1966) it is assumed that for all x:

$$\frac{\partial H}{\partial p_i} \neq 0 \qquad i = 1, \ldots, n \tag{36}$$

The theorem of section 3 has as an application the following singular variational problem. Suppose that

$$\frac{\partial H}{\partial p_i}(x_o,p) = 0, \text{ for all } i \tag{37}$$

and that $\partial H/\partial p_i$ vanishes nowhere else. Also suppose that we are given exactly one extremal through x_o and the value of ϕ on this extremal. The theorem of section 3 indicates that the variational problem will have a unique solution.

A certain trade-off in information is involved in this application. Namely, instead of (36), we require the knowledge of one extremal curve. Such problems have not received much attention in the calculus of variations. However, exactly this sort of problem arises in the asymptotic solution of certain diffusion equations (Ludwig, 1975), (Mangel, 1977), (Ventcel and Freidlin, 1970).

SECTION 5

AN APPLICATION

We conclude by showing how the theorem of section 3 applies to a special case of the problem posed by (1) and (2). We set $f(\psi) = \psi$. The initial data is $\psi = 0$ on S. We assume that Q is the singularity on S. Let $B = (b,{}^i{}_j)|_Q$; we assume that B has one real negative $(\lambda-)$ and one real positive (λ_+) eigenvalue. The manifold S is calculated by moving away from Q in the direction of the eigenvector of the negative eigenvalue and integrating

$$\frac{dx^i}{dt} = -b^i(x) \qquad i = 1, 2 \tag{38}$$

The problem as posed is characteristic. When (1) is differentiated with aspect to x^k and evaluated on S we obtain

$$b^i{}_{,k}\psi_i + b^i\psi_{ik} - \frac{a^{ij}}{2}\psi_i\psi_j\psi_k = 0 \tag{39}$$

or

$$\frac{d\psi_k}{dt} + b^i{}_{,k}\psi_i - \frac{a^{ij}\psi}{2}\psi_i\psi_j\psi_k = 0 \tag{40}$$

Equation (40) can be used to derive an equation for ψ_n (since $\psi_t = 0$ on S). We find that ψ_n satisfies

$$\frac{d\psi_n}{dt} + \hat{b}\psi_n - \frac{\hat{a}}{2}\,\psi_n^3 = 0 \quad \text{on } S \tag{41}$$

The functions $\hat{b}$, $\hat{a}$ are expressed in terms of the original b^i, (a^{ij}), (Mangel, 1977). In this case, Q corresponds to $t = \infty$.
At Q, $\frac{d\psi_n}{dt} = 0$ for all i, so that (41) yields

$$\hat{b}\psi_n(\infty) - \frac{\hat{a}}{2}\psi_n^{(\infty)} = 0$$

or

$$\psi_n(\infty) = \sqrt{\frac{2\hat{b}(\infty)}{\hat{a}(\infty)}} \tag{42}$$

Equation (41) is a form of Abel's equation and has the solution

$$\psi_n(t) = \left\{ \int_t^{\infty} \hat{a}(S)\exp\left[-2\int_t^{S} \hat{b}(S')dS'\right] dS \right\}^{-1/2} \tag{43}$$

We have constructed the unique normal derivative of ψ on S.

We now consider the ray equations. We switch to normal (N) and tangential (Y) coordinates (on S), with the origin at Q. In the vicinity of Q, the ray equations are

$$\frac{dN}{d\tau} = \lambda_+ N - a^{NN}p_N\psi + 0(N^2 + Y^2) \tag{44}$$

$$\frac{dY}{d\tau} = \lambda_- Y - a^{NN} p_N \psi + 0(N^2 + Y^2) \tag{45}$$

$$\frac{dx}{d\tau} = -\frac{1}{2} a^{NN} p_N \psi + 0(N^2 + Y^2) \tag{46}$$

$$\frac{dp_k}{d\tau} = - b^i{}_k p_i + \frac{a^{ij}{}_k}{2} p_i p_j \psi + a^{ij} p_i p_j p_k \quad . \tag{47}$$

We introduce a new manifold S_γ , defined by $\psi = \gamma$. S_γ is parametrized by

$$S^1_\gamma = \{(N, Y): \ N = \gamma/\psi_N(Y) + o(\gamma^2)\} \tag{48}$$

On S_γ , equations (44,46) become

$$\left.\frac{dN}{d\tau}\right|_{\tau=0} = \frac{\lambda_+ \gamma}{N} - a^{NN} p^0_N \gamma + o(\gamma^2) \tag{49}$$

$$\left.\frac{d}{d\tau}\right|_{\tau=0} = -\frac{1}{2} a^{NN} p^0_N \gamma + o(\gamma^2) \tag{50}$$

We reparametrize by $\sigma = \tau\gamma$ and find

$$\lim_{\gamma\to 0} \left.\frac{dN}{d\sigma}\right|_{\sigma=0} = \frac{\lambda_+}{\psi_N} - a^{NN} p^0_N \neq 0 \tag{51}$$

$$\lim_{\gamma\to 0} \left.\frac{dx}{d\sigma}\right|_{\sigma=0} = -\frac{1}{2} a^{NN} p^0_N \neq 0 \tag{52}$$

Hence, the ray equations (51, 52, 45, 47) can be integrated to determine $\psi(x)$ off of S .

Other examples of such characteristic initial value problems are given in Mangel (1977).

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